O LEVEL MATHEMATICS

WORKBOOK

BASED ON THE NEW LOWER SECONDARY CURRICULUM



Key Topics covered in the book

- NUMBER BASES
- WORKING WITH INTEGERS
- FRACTIONS, PERCENTAGES AND DECIMALS
- RECTANGULAR CARTESIAN COORDINATES IN 2 DI-MENSIONS
- GEOMETRIC CONSTRUCTION SKILLS
- SEQUENCE AND PATTERNS
- BEARINGS
- GENERAL AND ANGLE PROPERTIES OF GEOMET-RIC FIGURES
- DATA COLLECTION AND PRESENTATION .
- REFLECTION
- EQUATION OF LINES AND CURVES
- TIME AND TIME TABLES

NEW EDITION 2020

The only way to learn mathematics is to do mathematics.

Preface

Due to the increased demand of my textbooks by the learners, teachers and parents ,i have come up with this new edition of mathematics work book to supplement on the already designed learners book by the NCDC .My most important objective has been to make the book more user friendly.To accomplish this i have used simple language and i have included many new features that will help students learn from the book.Supplementary notes have been included for topics that hadnot been covered by learners by the time schools were closed.

The book contains exhaustive explanatory notes that are aimed at equiping students with mathematical skills.Questions have been provided on topics such as Number bases,working with integers and many others .I am convinced that any student who will use this book will never remain the same

ACKNOWLEDGEMENTS

First of all I offer my sincere gratitude to the almighty God, who has brought me this far because with his grace anything is possible. I would firstly like to thank my friends and teachers more so,Dr Henry Busulwa(Lecturer Makerere university), Dr Tadeo Senyoga(Lecturer Makerere university),Mr Madoi geofrey,Fr Kyazze Frank(Nyenga seminary),Mr sekirabi edward (Nyenga seminary),Mr Makumbi edward(Ndejje ss),Mr waninga Robert (Bishop ss Mukono),Mr kamala Alfred (Gombe ss),Mr Nshuguyika stephen(Mt st mary's Namagunga),Mr ssempagala solomon(Namilyango high school), and Sylvia for your love, time and constant advice you shared with me during my studies . Thank you for the many helpful discussions, for your constant interest and time, encouragement and guidance . Really, you have both been amazing and an inspiration.Thank you so much am really indebted.

NOTE: This partial version of book is free and can be accessed through any means , however the full package is to be out soon , which will contain all the topics and assessment forms.

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Chapter 1: NUMBER BASES

1.0.1 Identifying numbers of different bases on an abacus

1. Which possible base does each abacus below represent?



2. Complete the table below

BASE	NUMERALS
four	
five	
six	
seven	
eight	
nine	
ten	
eleven	
twelve	
sixteen	

1.0.2 Place Values Using the Abacus

- 3. Represent the following numbers on an abacus
 - (i) 334_{five}

(ii) 122_{four}

(iii) 2120_{three}

(iv) 4501_{six}

(v) 5231_{ten}

(vi) 273_{eight}

(vii) 101_{two}

(viii) 74_{nine}

(ix) $t21_{eleven}$

4. State the place value of each numeral in the following numbers

(i) $t221_{eleven}$

(ii) 64_{seven}

(iii) 8441_{nine}

(iv) 342_{five}

- 5. State the value of each numeral in the following numbers:
 - (i) 3234_{five}

(ii) 1223_{four}

(iii) 21_{three}

(iv) 452_{six}

(v) 5239_{ten}

(vi) 11_{two}

1.0.3 Converting Numbers

NOTE:

- The digits 10 and 11 are represented by t and e respectively in number bases.For digits above 12 ,we can use any letter of your choice to represent them
- When converting a decimal number to base ten, the digits on the left of the decimal point are given positive powers starting from $0, 1, 2, 3 \cdots$, while the digits on the right of the decimal point are given negative powers starting from $-1, -2, -3 \cdots$

- 6. convert the following figures to base ten
 - (i) 343_{five}

(ii) $87t2_{eleven}$

(iii) $6et_{twelve}$

(iv) 24.43_{five}

(v) 0.454_{six}

- 7. Convert the following numbers to the bases indicated:
 - (i) 21_{ten} to the binary system

(ii) 5213_{ten} to the octal system

(iii) 5432 to base six

(iv) 342 to base eight

(v) 15529_{ten} to twelve

(vi) 234_{five} to base six

(vii) 561_{seven} to base nine

(viii) 654_{six} to base four

- 8. Find the value of n in the following equation
- (i) $42_n = 1010_{three}$

(ii) $205_n = 124_{nine}$

(iv) $1n1_{four} = 41_{six}$

(v) $45_n = 29$

(vii) $100001_{two} = 45_n$

1.0.4 Arithmetic Operation on Numbers in Various Bases

ADDITION

If the sum of the digits exceeds the base, divide that sum by the base then write down the remainder and carry the whole number.

9. Workout the following leaving your answer in the base indicated

(a) $123_{seven} + 542_{seven}$

(b) $211_{five} + 234_{five}$

(c) $32.24_{nine} + 5.16_{nine}$

10. Workout $121_{three} + 325_{seven}$ giving your answer in base five

SUBTRACTION:

In case of borrowing the new value is the sum of the base and the digit which was small.

- 11. Workout the following leaving your answer in the base indicated
 - (a) $63_{eight} 52_{eight}$

(b) Subtract 342_{eight} from 567_{eight}

(c) $t45_{twelve} - 376_{twelve}$

(d) $30.241_{five} - 14.143_{five}$

(e) Workout $221_{three} - 101_{two}$ giving your answer in base four Hint: First convert 221_{three} and 101_{two} to base ten and then finally express the answer in the required base

MULTIPLICATION

In multiplication , we first find the product of any two numbers as we do in base ten, then we divide this product by the base number and there after we write the remainder and carry the quotient to the next place value position.

- 12. Workout the following leaving your answer in the base indicated
 - (a) $136_{eight} \times 4_{eight}$

(b) $152_{eight} \times 43_{eight}$

(c) $et5_{twelve} \times 8t_{twelve}$

(d) $124_{five} \times 32_{five}$

(e) $10010_{two} \times 110_{two}$

(f) $675_{nine} \times 6_{nine}$

(g) Multiply 323_{five} by 23_{five}

(h) Workout $1011_{two} \times 12_{three}$ giving your answer in binary base Hint:First convert 1011_{two} and 12_{three} to base ten and then finally express the answer in the required base

DIVISION

STEPS

- change each number base to base ten
- Divide the two numbers in base ten
- change the result back to the required base
- 13. Workout the following leaving your answer in the base indicated
 - 1 $2212_{three} \div 21_{three}$ Hint: First convert 2212_{three} and 21_{three} to base ten and then finally express the answer in the required base

(a) $110111_{two} \div 101_{two}$

(b) $245_{eight} \div 5_{eight}$

(c) $1111_{two} \div 101_{two}$

(d) Divide 100011_{two} by 111_{two}

(e) Divide 150_{nine} by 20_{nine}



Coronavirus disease (COVID-19) is an infectious disease caused by a new virus. The disease causes respiratory illness (like the flu) with symptoms such as a cough, fever, and in more severe cases, difficulty breathing. The virus that causes COVID-19 is mainly transmitted through droplets generated when an infected person coughs, sneezes, or exhales. These droplets are too heavy to hang in the air, and quickly fall on floors or surfaces.

By March 19, 33 African countries had reported more than 600 cases and 17 deaths due to COVID-19, the disease caused by the new coronavirus. Some of the African countries that have reported cases of the disease include ,Rwanda,Tanzania,Kenya, Ethiopia, Sudan , Guinea, Morocco, Tunisia, Egypt, Algeria, Senegal, Togo, Cameroon, Burkina Faso, the Democratic Republic of the Congo (DRC), South Africa, Nigeria, Ivory Coast, Gabon and Ghana. In Africa the number of people who have gotten infected of COVID-19(corona virus) is increasing day and night, and its approximated to be about 633 cases. The following data has been gathered for some of the african countries.

		-
COUNTRY	INFECTED NUMBER	QUARANTINE PATIENTS
UGANDA	0	2
KENYA	8	31
TANZANIA	19	10
SENEGAL	37	254
DRC	12	24
EGYPT	126	240

Table 1.1: Table showing the trend of corona virus in some of the african countries(19th- 03-2020)

TASK INSRUCTION: Have your answers in the spaces provided

1. As a TIS student, how many Quaratine patients do we have in Africa when you are using the nonary base system?

2. As S.1 student, of TIS how many people are infected with the corona virus in Africa using the duodecimal base system ?

3. If you are sent to Egypt to make research, and you find out that Egypt is using the seximal base system , what would be the number of infected patients in Egypt?

4. According to research it has been found out that quarantine patients in senegal and DRC have been put in groups of five. As a student what is the total number of quarantine patients when arranged in groups of five?

5. How many people are infected with the corona virus in East african countries when using the system of 4 patients?

6. On 18th march 2020 Italy had registered approximately 1240 people who had been infected with corona virus ,unfortunately on 19th march 2020, 400 people died of the virus.?

(i) As a math student how many deaths did italy register on 19th march when using the nonary base system.

(ii). Using the system of 5 patients how many infected individuals did italy remain with on 19th march 2020?

7. In not more than 150 words, advice the ministry of Health of Uganda on how they can curb the spread of the disease to the different parts of the country.

Activity of intergration on number bases

On April 4, 2020 the Covid19 task force started the distribution of food in Kawempe Division(kampala district). Each member in the household was given a package containing 6 kgs of maize flour, and 3 kg of beans. There are 10 households in the community with 3, 5, 7, 4, 6, 5, 8, 12, 13, 4 members respectively. **TASK**

1. Determine the number of packages the task force distributed in kawempe division.

2. Determine the total weight of the maize flour that was distributed in the division. Practice makes mathematics easier

- 3. In case there are some remaining packages, discuss what the task force should do with them.
- 4. The prices of, beans and maize flour was approximated to be at 4000UGX and 2500UGX perkilogram respectively. What is the total amount of money spent by the government on maize flour and beans in the 10 households.

Chapter 2: WORKING WITH INTEGERS

2.0.1 Natural Numbers

Introduction

Natural numbers can be classified into various groups of numbers. In your primary education, you learnt numbers such as even, odd, prime and composite.

1. Natural Numbers

These are numbers used in counting.e.g $N = \{1, 2, 3, 4 \dots \}$

2. Whole Numbers

These are counting numbers including zero.e.g $W = \{0, 1, 2, 3, 4 \cdots \}$

3. Square Numbers

These are numbers got after multiplying a natural number by itself.e.g $S = \{1, 4, 9, 16 \cdots \}$

4. Cube Numbers

These are numbers got after multiplying a natural number three times.e.g $C = \{1, 8, 27, 64 \cdots \}$

5. Even Numbers

This is a number that is exactly divisible by two .e.g $E = \{2, 4, 6, 8 \cdots \}$

6. Factors

Factors of numbers are all numbers that divide exactly into it.e.g 6 is divisible $\{1, 2, 3, 6\}$ Therefore the factors of 6 are $F_6 = \{1, 2, 3, 6\}$

7. Prime Numbers

This is a number with only two factors one and itself .e.g $E = \{2, 3, 5, 7 \cdots \}$

8. Prime Factor

This is a factor which is a prime number.

9. Prime Factorisation

Expresses a number as a product of only its prime factors .

10. Composite Numbers

This is a number with more than two factors .e.g $\{4, 6, 8\cdots\}$

11. Multiple of a Number

Is that number multiplied by another integer .i.e When two numbers are multiplied together, the product is called multiple.g Multiples of 5 include $\{5, 10, 15, 20 \cdots \}$.

- 12. Lowest common multiple(L.C.M) Is the lowest multiple of two or more numbers
- 13. Highest Common Factor(H.C.F) Is the highest number that divides exacly in two or more numbers .H.C.F is also called Greatest Common Divisor(G.C.D)

2.0.2 Identify Even, Odd, Prime and Composite Numbers



Color the	numbers.
-----------	----------

Prime			Compo	site	ſ	Veither	prime	nor cor	nposite
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Writing and reading numbers

- 1. Write the following in words:
 - (a) 6,800
 - (b) 9,888,008

(c) 722,820,060

(d) 8,999,909,700

(e) 6,745,842,003

(f) 3,730,284,654,040

2. Write the following in figures

- (a) seven hundred three million seven thousand and six
- (b) Four billion seventy-nine million five thousand six
- (c) One trillion three hundred forty billion seven hundred seventy-five million two hundred sixty thousand
- (d) Nine hundred ninety- nine trillion seven hundred eighty eight billion five hundred ninety nine million nine hundred ninety nine thousand eight hunded eighty six
- (e) seventy seven million two hundred sixty seven thousand nine hundred eighty

2.0.3 Differentiating between natural numbers and whole numbers/integers

Activity: Relating natural numbers and integers

Two learners Hannah and Ritah went to the school canteen to buy some snacks for their breakfast. Ritah bought 3 pancakes at UGX.200 each and 1 ban at UGX. 300. Hannah checked her bag and found out that her money was stolen. She borrowed some money from Ritah. She bought four 4 pancakes and 2 bans. Questions

- (a) Which of the two learners had more money?
- (b) How much money did Hannah borrow from Ritah?
- (c) Ritah said that Hannah had negative UGX. 1400. Was she correct?

(d) Give reasons for your answer.

2.0.4 Use Directed Numbers (Limited to Integers) in Real-life Situations

Activity: Integers in real-life situations

Read the story below and answer the questions.

Once upon a time, there lived an old woman. She had hot and cold stones and a big pot of water. If she put one hot stone in the water, the temperature of the water would rise by 1 degree. If she took the hot stone out of the water again, the temperature would go down by 1 degree.i.e if the temperature of the water was 28 degrees and the old woman removes 2 hot stones ,the temperature would drop to 26 degrees ,and if the temperature of the water was at 85 degrees and the old woman adds 4 hot stones,the temperature would rise to 89 degrees.



Questions

- (a) If the temperature of the water is 24 degrees and the old woman adds 5 hot stones, what is the new temperature of the water?
- (b) Now imagine that the temperature of the water is at 29 degrees. The old woman takes a spoon and takes out 3 of the hot stones from the pot.What is the temperature of the water when the old woman removes 3 hot stones? Explain your answer.
- (c) The old woman also had cold stones. If she adds 1 cold stone to the water, the temperature goes down by 1 degree. The temperature of the water was 26 degrees. Then the old woman added 4 cold stones. What is the temperature of the water after the old woman added 4 cold stones

- (d) Give a reason for your answer.
- (e) Imagine that the temperature of the water was 22 degrees and the old woman removes 3 cold stones. What happens to the temperature of the water?
- (f) What is the new temperature of the water? Explain your answer.

2.0.5 Use the Hierarchy of Operations to Carry out the Four Mathematical Operations on Integers

- 3. Sarah moved 5 steps to the right from a fixed point. Then she moved 9 steps to the left.
 - (a) How far is Sarah from the fixed point?
 - (b) Peter gave his answer as 4 steps to the left of the fixed point and John as -4 (negative 4). Who is correct? Give reasons for your answer.

2.0.6 Use the Hierarchy of Operations to Carry out the Four Mathematical Operations on Integers

NUMBER LINE

Integers can be illustrated on a number line (number scale) as shown below



Positive integers are to the right of a zero and negative integers are to the left of zero.Positive integers are shifts to the right while negative integers are shifts to the left

ADDITION OF NUMBERS ON A NUMBER LINE Using a number line Workout

Work out -4 + 6 using a numberline



 $^{-4} + ^{+6} = ^{+2}$

SUBTRACTION OF NUMBERS ON A NUMBER LINE

Work out -3 - 4 using a number line



 $^{-3} + ^{-4} = ^{-7}$

MULTIPLICATION OF NUMBERS ON A NUMBER LINE

Multiplication is interpreted as repeated addition of positive or negative numbers. Work out $^{+}2 \times ^{+}3$

SOLUTION

 $+2 \times + 3 = + 2 + + 2 + + 2$



 $+2 \times + 3 = 6$

DIVISION OF NUMBERS ON A NUMBER LINE

Division is interpreted as repeated subtraction of positive or negative numbers STEPS

- Draw an empty number line.
- Start from the right hand side of the number line.i.e From the Dividend

 $^{+}6$

- Subtract by groups i.e subtract the divisor from the dividend up to when you reach zero.
- Count the jumps made from the dividend
- (i) Workout $+6 \div^+ 2$ using a numberline

$$^{+}6 = 6 - 2 - 2 - 2$$

= 0
 $\div^{+} 2 = 3$

3 is the number of times you can subtract 2 from 6 before you get to zero i.e 3 represents the jumps made (skip 2 digits from 6)

(ii) Work out $40 \div 8$

Dividend

$$\downarrow$$

 $40 \div 8 = 5$
Divisor Quotient

RULE	RESULT	EXAMPLE
A positive \times A positive	A positive	$+2 \times + 4 = + 8$
A positive \times A negative	A negative	$^{+}2 \times ^{-}3 = ^{-}6$
A negative \times A negative	A positive	$^{-2} \times ^{-2} = ^{+4}$
A positive \div A positive	A positive	$+4 \div + 2 = + 2$
A positive \div A negative	A negative	$+4 \div 2 = 2$
A negative \div A positive	A negative	$-4 \div^+ 2 = -2$
A negative ÷A negative	A positive	$-4 \div 2 = 2$

Summary for the rules of multiplication and division of integers

4. Workout the following numbers using a number line

(a) +3 + 5

(b) +4-6

(c) $^{-7}-^{-3}$

(d) +4 - +8

(e) +5 - 3

(f) $4 \times + 2$

(g) $+3 \times -3$

(h) $^{+}4 \times ^{-}2$

(i) $+23 \times + 6$

(j) +3 -- 4

(k) $+9 \div 3$

(l) $24 \div 4$

5. Work out the following in degrees



- (c) $\ ^{-}17 \ ^{-}13$
- (d) +104 +5
- (e) +51 32

(f)	$42 \times^+ 2$
(g)	$^{+13} \times ^{-3}$
(h)	$^{+74} \times^{-2}$
(i)	$^{+123 \times + 6}$
(j)	$^{+}73 - ^{-}4$

(k) $+99 \div 3$

(l) $124 \div 4$

- 7. Work out Hint:BODMAS MUST BE APPLIED
 - (a) $+3 \times 4 \times 6$

(b) $^{+}4 \times ^{-}2 \times ^{+}5$

(c)
$$+7 \times -8 \times +4$$

(d) $^{-}20 \times ^{-}6 \div ^{+}2$

(e) $^{-}25 \div 5 \times ^{-}8$

(f) $^{-}34 \times ^{+}2 \div ^{+}2$

(g) 24 of
$$13 - (18 \div 6 + 3) \div (9 \times 3 - 25)$$

(h) $89 - (99 - 84 \div 2 + 2)$

(i) $6 \div (2 + (2 \times 6 - 2))$

(j) 4 of $(4+3) - 2(1+9) \div 4$

8. In a certain mathematics test a correct answer scores 5 marks and an incorrect answer, the child gets a penalty of two marks deducted. Joy guessed all the answers. She got 12 correct and 8 wrong. Work out her total marks.

2.0.7 Finding the Factors and Multiples of numbers

- 9. List all the common divisors/ factors of the following:
 - (a) 16
 - (b) 60
 - (c) 112
 - (d) 225
- 10. List down multiples of the following numbers that are less than 50
 - (a) 5
 - (b) 20
 - (c) 9
 - (d) 10

2.0.8 Finding the Prime Factors of any Number

Express each of the following numbers as a product of its prime factors

(a) 36

Prime Factor	Number	
2	36	
2	18	
3	9	
3	3	
	1	
	36	$5 = 2 \times 2 \times 3 \times 3$
	36	$5 = 2^2 \times 3^2$

NOTE

To express our answer in power notation we apply the law of indices as below

LAW	Example
$a^m \times a^n = a^{(m+n)}$	$2^3 \times 2^4 = 2^7$
$a^m \div a^n = a^{(m-n)}$	$2^5 \div 2^2 = 2^3$
$(a^m)^n = a^{m \times n}$	$(2^3)^3 = 2^9$
$a^1 = a$	$2^1 = 2$
$a^0 = 1$	$2^0 = 1$
$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$
$a^{-m} = \frac{1}{a^m}$	$2^{-4} = \frac{1}{2^4}$

(b) 108

Prime Factor	Number
2	108
2	54
3	27
3	9
3	3
	1

$$108 = 2 \times 2 \times 3 \times 3 \times 3 \tag{2.2}$$

(2.1)

$$108 = 2^2 \times 3^3 \tag{2.3}$$

\mathbf{NB}

Equation (2.2) expresses our answer as a product of prime factors Equation (2.3) expresses our answer in power notation The above method in equation(2.1) is known as prime factorisation

11. Find the prime factors of the following numbers. Give your answer in power form(Power notation).

(a) 28

(b) 54

(c) 204

(d) 156

(e) 225

2.0.9 Highest Common Factor(HCF)

STEPS

To find the HCF of two or more numbers:

- Express each of the numbers as a product of prime factors,
- Pick out the least power of each common factor. The product of these gives the HCF or GCF

EXAMPLES

(a) Find the HCF of 210 and 360

SOLUTION

Prime Factor	Number
2	210
3	105
5	35
7	7
	1

 $210 = 2^1 \times 3^1 \times 5^1 \times 7^1$

Prime Factor	Number
2	360
2	180
2	90
3	45
3	15
5	5
	1

 $360 = 2^3 \times 3^2 \times 5^1$

The common factors are 2,3 and 5. So we pick out those with the lowest(smaller) power . i.e On 2^3 and 2^1 , we choose the one with the smaller power , which is 2^1

$$HCF = 2^{1} \times 3^{1} \times 5^{1}$$
$$= 2 \times 3 \times 5$$
$$HCF = 30$$

METHOD 2

(b) Find the HCF of 12 and 15.

SOLUTION

 $F_{12} = \{1, 2, 3, 4, 6, 12\}$ and $F_{15} = \{1, 3, 5, 15\}$ The common factors are $\{1, 3\}$. The highest of these is 3. Therefore, the HCF of 12 and 15 is 3.

- 12. Find the HCF of the following:
 - (a) 96, 57

(b) 49,84

(c) 72,144,288

(d) 28,42,98

2.0.10 Work Out and Use Divisibility Tests of Some Numbers

13. Given the following numbers: 111,621,2821,10044,686,795,132,444,234,1089,485,12655756 TASK

Find out which of them are divisible by:

(a) 3
(b) 2

- (c) 4
- (d) 9
- (e) 11

2.0.11 Lowest Common Multiple(LCM)

(a) Find the LCM of 16, 12 and 24.

	16	12	24	
2	8	6	12	
2	4	3	6	
2	2	3	3	
2	1	3	3	
3	1	1	1	
		I	CM	$= 2 \times 2 \times 2 \times 2 \times 2 \times 3$
		Ι	CM	= 48

(b) Find the LCM of 210 and 360

SOLUTION

	210	360												
2	105	180												
2	105	90												
2	105	45												
3	35	15												
5	35	5												
7	7	1												
	1	1												
		LCM	= 2	2×2	$2 \times$	2	×	3	×	3	\times	5	\times	7
		LCM	= 2	2520)									

- 14. Find the LCM of the following numbers:
 - (a) 216, 288

(b) 120, 24, 18

(c) 30, 45, 80,65

(d) 225,105,210

15. Determine the smallest sum of money out of which a number of men, women and children may receive UGX. 150, Ush.50 and Ush.200 each.

Chapter 3: FRACTIONS, PERCENTAGES AND DECIMALS

Introduction

In this topic, you will use knowledge of place values to manipulate fractions, decimals and percentages. You will convert fractions to decimals, decimals to percentages and vice versa.

- 1. A fraction is a number in the form $\frac{a}{b}$
- 2. In a fraction the top number is called the numerator(a) and the bottom number is called the denominator(b)
- 3. A fraction is in simplest form (lowest terms) when the top and bottom cannot be any smaller
- 4. In comparing fractions the one with a larger percentage is the largest

3.0.1 Types of fractions

• Proper fraction

In a proper fraction the numerator is less than the denominator. Thus $\frac{3}{4}$ and $\frac{7}{8}$ are both proper fractions.

• Improper fraction

In an improper fraction the numerator is greater than the denominator. Thus $\frac{7}{5}$ and $\frac{4}{3}$ are both improper fractions.

• Equivalent fractions

Equivalent fractions have the same value. In an equivalent fraction both the numerator and denominator are multiplied or divided by the same number. Thus $\frac{5}{8}$ and $\frac{10}{16}$ are equivalent fractions

NOTE

In a mixed number a whole number is followed by a proper fraction. Thus $1\frac{3}{4}$ and $3\frac{5}{8}$ are both mixed numbers. A mixed number can be converted into an improper fraction and vice versa

1. Roberta shades $\frac{4}{7}$ of a shape. What fraction of the shape is left unshaded?

2. A tablet is divided into 4 equal parts. Stephen swallows $\frac{1}{4}$ of the tablet and Priscilla eats another $\frac{2}{4}$ of the tablet. What fraction of the tablet is left?

- 3. During a graduation party ,the graduand divided the cake into 12 equal parts. The visitors eat $\frac{5}{12}$ of the cake, the parents eat $\frac{1}{12}$ of the cake. What fraction of the cake is left
- 4. A rectangular field contains 40 spaces for car parking. There are 32 cars parked in the field p.
 - (a) What fraction of the car park is full?
 - (b) What fraction of the car park is empty?
- 5. Benjamin eats $\frac{3}{12}$ of the sweets in a packet. Shakur eats another $\frac{5}{12}$ of the sweets.
 - (a) What fraction of the sweets has been eaten?
 - (b) What fraction of the sweets is left?
- 6. Draw a circle with its two lines of symmetry.
 - (a) Shade $\frac{2}{4}$ of the shape.

- (b) Shade another $\frac{1}{4}$ of the shape.
- (c) What is the total fraction now shaded?
- (d) How much is left unshaded?

3.0.2 Converting Improper Fractions to Mixed Numbers and Vice Versa

Summary

$$\label{eq:main_state} \begin{split} \frac{(D \times W) + N}{D} \\ Where: \\ D = denominator \\ N = numerator \\ W = wholenumber \end{split}$$

EXAMPLES

1. Convert $3\frac{2}{5}$ into an improper fraction **SOLUTION**

$$\frac{(D \times W) + N}{D}$$

$$Where:$$

$$D = 5$$

$$N = 2$$

$$W = 3$$

$$\frac{(D \times W) + N}{D} = \frac{(5 \times 3) + 2}{5}$$

$$= \frac{15 + 2}{5}$$

$$= \frac{17}{5}$$

2. Express $\frac{11}{4}$ as a mixed number.

We are required to express our answer in the form $W_{\overline{D}}^{R}$

$$\frac{11}{4} = 2$$
remainder 3
$$= 2\frac{3}{4}$$

3. Reduce $\frac{5}{10}$ to its simplest form

$$\frac{5}{10} = \frac{5 \div 5}{10 \div 5}$$
$$= \frac{1}{2}$$

4. Find the equivalent fractions for $\frac{1}{3}$ We can find the equivalent fractions by multiplying the numerator and denominator by the same number

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$
$$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12}$$
$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15}$$
Therefore $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{5}{15}$

1. Convert these mixed numbers to improper fractions.

(a) $5\frac{4}{7}$

- (b) $15\frac{1}{8}$
- (c) $3\frac{7}{9}$
- (d) $1\frac{2}{4}$
- 2. Write these fractions in order of increasing size. $6\frac{1}{3}, \frac{8}{10}, frac1412, frac1315$

3. Arrange the fractions $2\frac{5}{6}$, $6\frac{4}{9}$, $\frac{7}{8}$ and $\frac{1}{2}$ in descending order of magnitude

4. A young child is 64 months old. Find the age of the baby in years as a mixed number in the simplest form.

5. In an office there are $5\frac{1}{5}$ reams of paper. There are 500 sheets of paper in each full ream. How many sheets of paper are there in the office?

- 6. Change these mixed numbers to vulgar fractions:
 - (a) $6\frac{3}{5}$
 - (b) $3\frac{2}{17}$
- 7. Express the following improper fractions as a mixed number.
 - (a) $\frac{38}{9}$
 - (b) $\frac{231}{15}$
 - (c) $\frac{54}{7}$

- 8. Draw diagrams to show these improper fractions:
 - (a) $\frac{9}{4}$

(b) $\frac{8}{3}$

3.0.3 Operations on Fractions

- For fractions with plus (+) and minus (-) signs only, find the LCM and workout
- For fractions with combined operations, the BODMAS rule must be observed.

Work out problems from real-life situations

EXAMPLES

1. Find $\frac{1}{10}$ of UGX. 10000

SOLUTION

$$= \frac{1}{10} \text{ of } 10000$$
$$= \frac{1}{10} \times 10000$$
$$= \frac{1}{10} \times 10000$$
$$= \text{UGX1000}$$

2. Find $\frac{4}{8}$ of UGX. 16,000

SOLUTION

$$= \frac{4}{8} \text{ of } 16,000$$

= $\frac{4}{8} \times 16,000$
= $\frac{4}{8} \times 16000^{-2000}$
= 4×2000
= UGX8000

1. Find:

(a) $\frac{1}{4}$ of UGX.24,000

(b) $\frac{1}{4}$ of 180

(c) $\frac{8}{7}$ of 56

(d) $\frac{5}{9}$ of 8100

(e) $\frac{7}{10}$ of UGX. 2,500,000

2. In a test, there are 80 marks. Mimmi gets $\frac{6}{8}$ of the marks. How many marks does she get?

3. At Taibah international school school there are 850 pupils. If $\frac{3}{50}$ of the pupils are left-handed, how many left-handed pupils are there in the school?

3.0.4 Addition of Fractions with the Same Denominators

To ADD fractions with like or the same denominator, simply add the numerators then copy the common denominator. Always reduce your final answer to its lowest term.



2. ADD $\frac{2}{9} + \frac{1}{9}$



- 1. Work out $\frac{4}{8} + \frac{2}{8}$
- 2. Work out $2\frac{1}{3} + 3\frac{2}{3}$
- 3. Work out $\frac{5}{12} + \frac{3}{12} + \frac{1}{12}$
- 4. If Praise sells $\frac{5}{8}$ of her clothes to Maria keziah, and $\frac{2}{8}$ of it to Alexadra. What fraction of her clothes has she sold.

5. Tendo ate $\frac{1}{3}$ of a fish for lunch and another $\frac{1}{3}$ of the fish for supper. What fraction of the fish did Tendo eat altogether

6. $\frac{5}{6} + \frac{2}{6} + \frac{7}{6}$

7. $1\frac{1}{3} + 3\frac{2}{3} + 5\frac{1}{3}$

3.0.5 Addition of Fractions with different Denominators

Given two unlike fractions where the denominators are NOT the same, the fractions can be solved using two methods.

- LCM method
- Cross Multiplication method

The following steps are followed when using the LCM method Steps for Adding Fractions with Unlike Denominators

- Identify the least common denominator by finding the least common multiple for the denominators.
- Write equivalent fractions (making sure that each equivalent fraction contains the least common denominator (LCM))
- Add the equivalent fractions that you wrote in step 2. (The denominators should now be the same.)
- Reduce the fraction to its lowest term

EXAMPLES

1. Add $\frac{3}{4} + \frac{1}{3}$ STEP 1:Finding the LCM of 4 and 3. LCM=12

Divide the denominator by the LCM and then multiply it with the numerator

$$\frac{3}{4} + \frac{1}{3} = \frac{(12 \div 4) \times 3 + (12 \div 3) \times 1}{12}$$
$$= \frac{3 \times 3 + 4 \times 1}{12}$$
$$= \frac{9 + 4}{12}$$
$$= \frac{13}{12}$$

2. Add $\frac{3}{5} + \frac{2}{9}$ METHOD 2



3. Add $\frac{1}{8} + \frac{1}{3}$ LCM method STEP 1:Finding the LCM of 8 and 3. LCM=24

Divide the denominator by the LCM and then multiply it with the numerator

$$\frac{1}{8} + \frac{1}{3} = \frac{(24 \div 8) \times 1 + (24 \div 3) \times 1}{24}$$
$$= \frac{3 \times 1 + 8 \times 1}{24}$$
$$= \frac{3 + 8}{24}$$
$$= \frac{11}{24}$$



4. Add $\frac{1}{2} + \frac{1}{3}$ STEP 1:Finding the LCM of 2 and 3. LCM=6

Divide the denominator by the LCM and then multiply it with the numerator

$$\frac{1}{2} + \frac{1}{3} = \frac{(6 \div 2) \times 1 + (6 \div 3) \times 1}{6}$$
$$= \frac{3 \times 1 + 2 \times 1}{6}$$
$$= \frac{3 + 2}{6}$$
$$= \frac{5}{6}$$

1. Work out $\frac{5}{7} + \frac{2}{3}$

2. Work out $\frac{3}{5} + \frac{2}{9}$

3. Work out $\frac{7}{8} + \frac{4}{6}$

Subtraction of Fractions with Same Denominators

To SUBTRACT fractions with like or the same denominator, simply subtract the numerators then copy the common denominator. Always reduce your final answer to its lowest term.



EXAMPLES

1. Work out $\frac{3}{4} - \frac{2}{4}$

SOLUTION



2. Work out $\frac{2}{9} - \frac{1}{9}$

$$\frac{2}{9} - \frac{1}{9} = \frac{2 - 1}{9} = \frac{1}{9}$$

1. Work out $\frac{5}{7} - \frac{2}{7}$

2. Work out $4\frac{5}{7} - 2\frac{2}{7}$

3. Kristi had $\frac{9}{10}$ of a cake , she ate $\frac{7}{10}$ of it. What fraction remained

Subtraction of Fractions with different Denominators

Given two unlike fractions where the denominators are NOT the same, we follow the same steps as in addition.

EXAMPLES

1. Work out $\frac{6}{11} - \frac{3}{22}$ STEP 1:Finding the LCM of 11 and 22. LCM=22 Divide the denominator by the LCM and then multiply it with the numerator

$$\frac{6}{11} - \frac{3}{22} = \frac{(22 \div 11) \times 6 - (22 \div 22) \times 3}{22}$$
$$= \frac{2 \times 6 - 1 \times 3}{22}$$
$$= \frac{12 - 3}{22}$$
$$= \frac{9}{22}$$

Cross multiplication method

$$\frac{6}{11} - \frac{3}{22} = \frac{(22 \times 6) - (11 \times 3)}{22 \times 11}$$
$$= \frac{132 - 33}{242}$$
$$= \frac{99}{242}$$
$$= \frac{99}{242}$$
$$= \frac{99}{242}^{-9}$$
$$= \frac{99}{242}^{-22}$$

2. Workout $\frac{5}{6} - \frac{1}{3}$ STEP 1:Finding the LCM of 6 and 3. LCM=6

Divide the denominator by the LCM and then multiply it with the numerator

$$\frac{5}{6} - \frac{1}{3} = \frac{(6 \div 6) \times 5 - (6 \div 3) \times 1}{6}$$
$$= \frac{1 \times 5 - 2 \times 1}{6}$$
$$= \frac{5 - 2}{6}$$
$$= \frac{3}{6}$$
$$= \frac{3}{6}$$
$$= \frac{3}{6}$$
$$= \frac{3}{6}$$
$$= \frac{3}{6}$$
$$= \frac{3}{6}$$

Cross multiplication method

$$\frac{5}{6} - \frac{1}{3} = \frac{(3 \times 5) - (6 \times 1)}{3 \times 6}$$
$$= \frac{15 - 6}{18}$$
$$= \frac{9}{18}$$
$$= \frac{9}{18}$$
$$= \frac{9}{18}$$
$$= \frac{9}{18}$$

- 1. Work out $\frac{7}{11} \frac{4}{22}$
- 2. Work out $4\frac{5}{6} \frac{1}{3}$
 - (a) Work out $\frac{3}{4} \frac{1}{2}$
 - (b) $\frac{5}{6} \frac{3}{4}$

(c) $\frac{3}{4} - \frac{1}{3}$

(d) $\frac{4}{5} - \frac{3}{6}$

Addition of Mixed Fractions

1. Work out $1 + \frac{3}{5}$

$$1 + \frac{3}{5} = 1\frac{3}{5}$$

2. Work out $5 + \frac{5}{9}$

$$5 + \frac{5}{9} = 5\frac{5}{9}$$

3. Work out $3 + \frac{7}{8}$

$$3 + \frac{7}{8} = 3\frac{7}{8}$$

4. Work out $1\frac{1}{2} + 2\frac{3}{5}$

Convert the mixed fractions into an improper fraction

$$= \frac{(2 \times 1) + 1}{2} + \frac{(2 \times 5) + 3}{5}$$
$$= \frac{3}{2} + \frac{13}{5}$$

Finding the LCM of 5 and 2 LCM=10

$$= \frac{(10 \div 2) \times 3 + (10 \div 5) \times 13}{10}$$

= $\frac{5 \times 3 + 2 \times 13}{10}$
= $\frac{15 + 26}{10}$
= $\frac{41}{10}$

1. Work out $1\frac{5}{7} + 4\frac{2}{7}$

2. Work out $6\frac{3}{4} + 4\frac{1}{3}$

3. Daniel bought $6\frac{1}{4}$ kg of chicken on monday and $7\frac{3}{4}$ kg on tuesday. How many kilograms did he buy altogether.

Subtraction of Mixed Fractions

1. Subtract $6\frac{5}{7} - 2\frac{1}{7}$

Changing the mixed fractions into an improper fraction

$$= \frac{47}{7} - \frac{15}{7} \\ = \frac{47 - 15}{7} \\ = \frac{32}{7} \\ = 4\frac{4}{7}$$

2. Subtract $4\frac{3}{5} - 2\frac{1}{5}$

Changing the mixed fractions into an improper fraction

$$= \frac{23}{5} - \frac{11}{5}$$
$$= \frac{23 - 11}{5}$$
$$= \frac{12}{5}$$
$$= 2\frac{2}{5}$$

1. Work out $3\frac{1}{5} - 1\frac{7}{8}$

2. Work out $6\frac{3}{4} - 4\frac{1}{3}$

Multiplication of Fractions

When multiplying fractions, the numerator and the denominator are multiplied separately.

1. Work out $\frac{3}{5} \times \frac{2}{7}$

$$\frac{\frac{3}{5} \times \frac{2}{7}}{\frac{2}{5} \times \frac{3 \times 2}{5 \times 7}} = \frac{\frac{6}{35}}{\frac{6}{35}}$$

2. Work out $\frac{6}{9}of\frac{3}{7}$

$$\frac{6}{9}of\frac{3}{7} = \frac{6}{9} \times \frac{3}{7}$$
$$= \frac{6 \times 3}{9 \times 7}$$
$$= \frac{18}{63}$$
$$= \frac{18^{*2}}{63^{*7}}$$
$$= \frac{2}{7}$$

1. Work out $\frac{2}{3} \times \frac{5}{7}$

2. Work out $\frac{3}{4} \times \frac{8}{9}$

Multiplying Mixed Fractions

Convert the mixed fraction into an improper fraction ,and then apply the multiplication rules

1. Work out $4\frac{3}{5} \times 2\frac{1}{5}$

Changing the mixed fractions into an improper fraction

$$= \frac{23}{5} \times \frac{11}{5}$$
$$= \frac{23 \times 11}{5 \times 5}$$
$$= \frac{253}{25}$$
$$= 10\frac{3}{25}$$

2. Work $\operatorname{out}2\frac{3}{5} \times 2\frac{3}{7}$

Changing the mixed fractions into an improper fraction

$$= \frac{13}{5} \times \frac{17}{7}$$
$$= \frac{13 \times 17}{5 \times 7}$$
$$= \frac{221}{35}$$
$$= 6\frac{11}{35}$$

1. Work out $1\frac{7}{8} \times 3\frac{2}{7}$

2. Work out $8\frac{7}{5} \times 4\frac{2}{3}$

Division of Fractions

Flip And Multiply

1. Work out $\frac{6}{9} \div \frac{7}{3}$

Flip the second fraction and then multiply

$$\frac{6}{9} \div \frac{7}{3} = \frac{6}{9} \times \frac{3}{7}$$
$$= \frac{6 \times 3}{9 \times 7}$$
$$= \frac{18}{63}$$
$$= \frac{18^{2}}{63^{2}}$$
$$= \frac{18^{2}}{7}$$

2. Work out $\frac{7}{8} \div \frac{4}{3}$

Flip the second fraction and then multiply

$$\frac{7}{8} \div \frac{4}{3} = \frac{7}{8} \times \frac{3}{4}$$
$$= \frac{7 \times 3}{8 \times 4}$$
$$= \frac{21}{32}$$

1. Work out $\frac{3}{8} \div 2\frac{1}{4}$

2. Work out $\frac{4}{7} \div 2\frac{1}{3}$

3. Work out $21 \div \frac{7}{3}$

EXERCISE

- 1. Without using a calculator, express the following as a single fraction:
 - (a) $\frac{2}{5} + \frac{3}{7}$
 - (b) $1\frac{2}{3} + 2\frac{1}{4}$
 - (c) $4\frac{1}{6} + 1\frac{19}{20} 3\frac{1}{20}$
 - (d) $1\frac{6}{7} + 2\frac{4}{9}$
 - (e) $\frac{1}{2} \frac{3}{4} + \frac{5}{8} \frac{7}{16} + \frac{19}{32}$
- 2. Express the following fractions in their lowest term
 - (a) $\frac{13}{26}$
 - (b) $\frac{30}{72}$
 - (c) $\frac{17}{68}$
 - (d) $\frac{56}{22}$
 - (e) $3\frac{14}{84}$

- (f) $10\frac{4}{28}$
- (g) $\frac{909}{27}$
- 3. Evaluate the following fractions. Answers should be as simplest as possible.
 - (a) $2\frac{5}{14} \times \frac{7}{66} \times 4$ (b) $7\frac{1}{8} \times \frac{5}{16}$ (c) $3\frac{2}{3} \div \frac{2}{5}$ (d) $1 \div 6\frac{12}{30}$ (e) $\frac{2}{5} \div \frac{1}{2}$ (f) $\frac{2}{3} \times \frac{5}{7} \times \frac{21}{32}$ (g) $\frac{2}{3}$ of $\frac{4}{5}$ (h) $21 \div \frac{7}{9}$ (i) $1\frac{2}{7} \div 6$ (j) $5\frac{1}{4}$ of $10\frac{4}{5}$

3.0.6 Fractions and decimals

A decimal number is a number with a decimal point. Thus 1.56 is a decimal number

Activity: Convert Fractions to Decimals To convert a fraction to decimal, divide the numerator by the denominator. EXAMPLES

- 1. Using a calculator, convert the following fractions into decimal numbers:
 - (a) $\frac{1}{2}$

$$\frac{1}{2} = 1 \div 2$$
$$= 0.5$$

(b) $\frac{1}{4}$

$$\frac{1}{4} = 1 \div 4$$
$$= 0.25$$

(c) $\frac{3}{4}$

$$\frac{3}{4} = 3 \div 4$$
$$= 0.75$$

Activity: Convert Decimals to Fractions

A decimal number can be exact or inexact An exact decimal or terminating decimal is a decimal that ends. This decimal is converted into a fraction as follows:

- Divide the decimal by one to get $\frac{\text{Decimal}}{1}$
- Multiply both top and bottom by 10 for every number after the decimal point.e.g 0.2 ,it has one number after the decimal point so we shall multiply by 10 .for 1.25, it has two numbers after the decimal point so we multiply by 100

EXAMPLES

- 1. Convert the following decimals to fractions
 - (a) 0.5

$$= \frac{\text{Decimal}}{1}$$
$$0.5 = \frac{0.5}{1}$$

we have one number after the decimal point so we multiply the denominator and numerator by $10\,$

$$= \frac{0.5 \times 10}{1 \times 10}$$
$$= \frac{5}{10}$$
$$= \frac{\cancel{10}}{\cancel{10}^2}$$
$$= \frac{1}{2}$$

(b) 1.05

$$= \frac{\text{Decimal}}{1}$$
$$1.05 = \frac{1.05}{1}$$

we have two numbers after the decimal point so we multiply the denominator and numerator by 100

$$= \frac{1.05 \times 100}{1 \times 100}$$

= $\frac{105}{100}$
= $\frac{105^{21}}{100^{20}}$
= $\frac{21}{20}$

(c) 0.625

$$= \frac{\text{Decimal}}{1}$$
$$0.625 = \frac{0.625}{1}$$

we have three numbers after the decimal point so we multiply the denominator and numerator by 1000

$$= \frac{0.625 \times 1000}{1 \times 1000}$$
$$= \frac{625}{1000}$$
$$= \frac{625}{1000}^{*5}$$
$$= \frac{5}{8}$$

Addition and subtraction of decimals

To add or subtract two decimal numbers, line up the decimal points and then workout.

1. Without using a calculator, evaluate:

(a) 3.21 + 4.5



(b) 0.32 + 12.965 + 1.1

Adding Decimals							
<i>Example:</i> 0.32 + 12.965 + 1.1	<i>Example:</i> 51 + 14.02 + 2.1						
Line up the decimal points + 1.100 (Pad' with zeros 14.385	51.00 14.02 + 2.10 67.12 Change whole number to decimal						

- 2. Without using a calculator, evaluate:
 - (a) 8.97 2.82



(b)
$$76.3 - 34.1$$

Line up the	Line up the
decimal points	decimal points
↓	↓
76.3	4.321
<u>- 34.1</u>	<u>- 4.1</u>
42.2	0.221

Multiplication and Division of decimals

To multiply or divide two decimal numbers, express the decimal numbers in fractions and then workout.

EXAMPLES

1. Without using a calculator, evaluate: 0.5×0.08

Convert the decimals into fractions

$$0.5 \times 0.08 = \frac{0.5 \times 10}{1 \times 10} \times \frac{0.08 \times 100}{1 \times 100}$$
$$= \frac{5}{10} \times \frac{8}{100}$$

Multiply the numerators and denominators separately

$$= \frac{5 \times 8}{10 \times 100}$$
$$= \frac{40}{1000}$$

Reduce the fraction in its lowest term

$$= \frac{40^{-1}}{1000^{-25}}$$
$$= \frac{1}{25}$$

2. Without using a calculator, evaluate: 0.25×0.004

Convert the decimals into fractions

$$0.25 \times 0.004 = \frac{0.25 \times 100}{1 \times 100} \times \frac{0.004 \times 1000}{1 \times 1000}$$
$$= \frac{25}{100} \times \frac{4}{1000}$$
cators and denominators separately

Multiply the numerators and denominators separately

$$= \frac{25 \times 4}{100 \times 1000} \\ = \frac{100}{100000}$$

Reduce the fraction in its lowest term

$$= \frac{100^{-1}}{10000^{-1000}}$$
$$= \frac{1}{1000}$$

3. Without using a calculator, evaluate: $\frac{0.032}{0.16}$

Convert the decimals into fractions

$$\frac{0.032}{0.16} = \frac{32}{1000} \div \frac{16}{100}$$

Flip the second fraction

$$= \frac{32}{1000} \times \frac{100}{16} \\ = \frac{32 \times 100}{1000 \times 16} \\ = \frac{3200}{16000} \\ = \frac{1}{5}$$

EXERCISE

- 1. Without using a calculator, evaluate: 13.79 12.547
- 2. Without using a calculator, evaluate: 136 14.54
- 3. Convert the following decimal numbers into fractions in their lowest terms:
 - (a) 0.125
 - (b) 0.08
 - (c) 0.75
 - (d) 0.375
 - (e) 0.625
 - (f) 1.75
 - (g) 2.35
 - (h) 0.3
 - (i) 0.37
 - (j) 0.0225
- 4. Without using a calculator, evaluate: 308.6 + 20.475 + 1.36
- 5. Using a calculator, convert the following fractions into decimal numbers:
 - (a) $\frac{5}{8}$
 - (b) $\frac{7}{4}$
 - (c) $\frac{47}{20}$
 - (d) $\frac{3}{8}$
 - (e) $\frac{17}{50}$

3.0.7 Identify and Classify Decimals as Terminating, Non-terminating and Recurring Decimals

- A terminating decimal (An exact decimal): is a decimal number that contains a finite number of digits after the decimal point. Fractions like $\frac{3}{5}, \frac{1}{2}, \frac{3}{8}$ can be converted into decimals and they end or terminate: $\frac{3}{5} = 0.6, \frac{1}{2} = 0.5, \frac{3}{8} = 0.375$
- Non-terminating decimal: is a decimal number that never repeats. Example : $0.076923 \cdots, 0.05882352 \cdots, 1.4223213345 \cdots$
- Recurring Decimal(Repeating decimals): is a decimal number that contains an infinite number of digits. Fractions like $\frac{2}{3}, \frac{2}{15}, \frac{1}{11}$ do not end or terminate when converted into decimals: $\frac{2}{3} = 0.666666 \cdots, \frac{2}{15} = 0.13333 \cdots, \frac{1}{11} = 0.09090 \cdots$

EXERCISE

- 1. Using a calculator Write the following fractions as recurring decimals:
 - (a) $\frac{36}{99}$
 - (b) $\frac{2}{11}$
 - (c) $\frac{1}{6}$
 - (d) $\frac{45}{99}$
 - (e) $\frac{5}{9}$

3.0.8 Converting Recurring Decimals into Fractions

- A recurring decimal is a decimal with endless repeating digits after the decimal point.
- A recurring decimal $0.363636\cdots$ is the same as $0.\overline{36}$ or $0.\dot{3}\dot{6}$
- A recurring decimal is converted into a fraction as follows:
 - 1 Let x = recurring decimal.
 - 2 Let n = the number of recurring digits
 - 3 Multiply the recurring decimal by 10^n E.g when only one number is repeating i.e $n=1,10^1 = 10$ so we multiply through out by 10,when $n=2,10^2 = 100$,so we multiply through out by 100
 - 4 Eliminate the recurring part by subtracting (3)-(1)
 - 5 Solve for x, expressing your answer as a fraction in its lowest form

EXAMPLES

1. Convert $0.5555 \cdots$ into a fraction.

SOLUTION

Let the fraction be x

$$x = 0.5555 \cdots \tag{3.1}$$

n=1,since we have only one repeating digit i.e 5 so 10^n , $10^1 = 10$ Multiply through Equation (1) by 10

$$x \times 10 = 0.5555 \dots \times 10 \tag{3.2}$$

$$10x = 5.555\cdots$$
 (3.3)

Subtracting Equation (3)-Equation(1)

(3.4)

$$\begin{cases} 10x = 5.555 \cdots \\ x = 0.555 \cdots \\ 9x = 5 \end{cases}$$
$$\frac{9x}{9} = \frac{5}{9}$$
$$\frac{\cancel{9x}}{\cancel{9}} = \frac{5}{9}$$
$$x = \frac{5}{9}$$

Divide through out by 9

2. Express $0.363636\cdots$ as a fraction in its simplest form

SOLUTION

 $-\begin{cases} 100x = 36.363636\cdots \\ x = 0.363636\cdots \end{cases}$

99x = 36

Divide through out by 99

$$\frac{99x}{99} = \frac{36}{99}$$
$$\frac{99x}{99} = \frac{36}{99}$$
$$x = \frac{36}{99}$$
$$x = \frac{4}{11}$$

3. Express $0.\overline{891}$ as a fraction in its simplest form

SOLUTION

$$\begin{cases} 1000x = 891.891891\cdots \\ x = 0.891891\cdots \\ 999x = 891 \\ \frac{999x}{999} = \frac{891}{999} \\ \frac{999x}{999} = \frac{891}{999} \\ \frac{999x}{999} = \frac{891}{999}^{37} \\ x = \frac{33}{37} \end{cases}$$

Divide through out by 999

4. Express
$$1.\dot{2}\dot{7}$$
 as a fraction in its simplest form

SOLUTION

Subtracting Equation (3)-Equation(1)

$$\begin{cases} 100x = 127.2727\cdots \\ x = 1.2727\cdots \\ 99x = 126 \\ \frac{99x}{99} = \frac{126}{99} \\ \frac{99x}{99} = \frac{126}{99}^{14} \\ \frac{99x}{99} = \frac{126}{99}^{11} \\ x = \frac{14}{11} \end{cases}$$

Divide through out by 99

5. Express $0.1\dot{6}$ as a fraction in its simplest form

SOLUTION

NOTE: For this question on the Right Hand Side(RHS) of the decimal point we have only one digit that is recurring, so for us to solve it we ought to remain with the recurring part on the RHS of the decimal point

Let the fraction be x

Both sides of the equation are multiplied by 10 (Since we have only one number after the decimal point that is not recurring) so that the repeating part of the number is immediately next to the decimal.

 $x \times 10 = 0.1666 \cdots \times 10$

n=1, since we have only one repeating digit i.e 6 so 10^n , $10^1 = 10$

Multiply through Equation (2) by 10

 $10x \times 10 = 1.666 \cdots \times 10$

Subtracting Equation (3)-Equation(2)

 $-\begin{cases} 100x = 16.666 \cdots \\ 10x = 1.666 \cdots \end{cases}$ 90x = 15 Divide through out by 90

$$\frac{90x}{90} = \frac{15}{90}$$
$$\frac{90x}{90} = \frac{15}{90}$$
$$x = \frac{15}{6}$$

6. Express $2.014545\cdots$ as a fraction in its simplest form

SOLUTION

NOTE: For this question on the Right Hand Side(RHS) of the decimal point we have only one digit that is recurring, so for us to solve it we ought to remain with the recurring part on the RHS of the decimal point

Let the fraction be x

Both sides of the equation are multiplied by 100 (Since we have two numbers after the decimal point that are not recurring) so that the repeating part of the number is immediately next to the decimal.

 $x \times 100 = 2.014545 \cdots \times 100$

n=2, since we have only two repeating digits i.e 4 and 5 so 10^n , $10^2 = 100$

Multiply through Equation (2) by 100

 $100x \times 100 = 201.4545 \cdots \times 100$

Subtracting Equation (3)-Equation(2)

$$-\begin{cases} 10000x = 20145.4545\cdots \\ 100x = 201.4545\cdots \end{cases}$$

9900x = 19944

Divide through out by 9900

$$\frac{9900x}{9900} = \frac{19944}{9900}$$
$$\frac{9900x}{9900x} = \frac{19944}{9900}^{554}$$
$$x = \frac{554}{275}$$

EXERCISE

- 1. Convert the following recurring decimals into fractions
 - (a) 0.777...
 - (b) 0.4444...
 - (c) 0.1333...
 - (d) $1.2565656 \cdots$
 - (e) 0.0131313...
- 2. Convert the following numbers into recurring decimals
 - (a) $\frac{7}{9}$
 - (b) $\frac{1}{3}$
 - (c) $\frac{2}{6}$
 - (d) $\frac{15}{99}$
- 3. Express the recurring decimal $1.633\cdots$ as a fraction
- 4. Convert $2.13535\cdots$ to a fraction.
- 5. Express $0.\dot{3}\dot{8}$ as a fraction in its simplest form
- 6. Express $2.\overline{43}$ as a fraction in its simplest form
- 7. Express $0.\overline{45}$ as a fraction in its simplest form
- 8. Express $0.\overline{63}$ as a fraction in its simplest form
- 9. Express $0.3\dot{7}$ as a fraction in its simplest form
- 10. Express $0.3181818 \cdots$ as a fraction in its simplest form

3.0.9 Percentages

- Percentage is a fraction whose denominator is 100.
- The Symbol for percentage is written as %.

Convert Fractions and Decimals into Percentages and Vice Versa

• To change a percentage into a fraction or decimal divide by 100. Thus

Percentage	Fraction	Fraction in lowest term	Decimal
60%	$\frac{60}{100}$	$\frac{3}{5}$	0.6
75%	$\frac{75}{100}$	$\frac{3}{4}$	0.75
15%	$\frac{15}{100}$	$\frac{3}{20}$	0.15

• To change a fraction into a percentage multiply by 100. Thus

Fraction	Conversion	Percentage			
$\frac{3}{5}$	$\frac{3}{5} \times 100$	60%			
$\frac{3}{4}$	$\frac{3}{4} \times 100$	75%			
$\frac{1}{2}$	$\frac{1}{2} \times 100$	50%			

• To change a decimal into a percentage multiply by 100. Thus

Decimal	Conversion	Percentage
0.5	0.5×100	50%
0.84	0.84×100	84%
0.125	0.125×100	125%

EXERCISE

- 1. Express each percentage as a fraction in its simplest form
 - (a) 16%
 - (b) 30%
 - (c) 24%
 - (d) 15.5%

2. Express each percentage as a decimal

- (a) 67%
- (b) 25%
- (c) 84.5%
- (d) 50%
- 3. Express each fraction as a percentage
 - (a) $\frac{9}{10}$
 - (b) $\frac{49}{50}$
 - (c) $\frac{11}{20}$
 - (d) $\frac{14}{25}$
- 4. Express each decimal as a percentage
 - (a) *o*.25
 - (b) 0.125
 - (c) 0.486
 - (d) 0.34
- 5. Find 10% of 40,000
- 6. Find 25% of 120

- 7. Express 44 as a percentage of 80
- 8. In a tray of 30 eggs 3 are rotten. Find the percentage of rotten eggs
- 9. In a mathematics test marked out of 60,Daniel obtained 56 marks. Find his percentage mark
- 10. In a class, 90 students are boys and 25% are girls. Find the number of girls in the class

3.0.10 Calculate a Percentage of a Given Quantity

The percentage of a quantity can always be calculated in terms of percentage increase or percentage decrease. Thus this is referred to as a percentage change

- Percentage change = $\frac{\text{Change in value}}{\text{Original value}} \times 100$
- Change in value = |New value-Old value|
- Percentage increase = $\frac{\text{increase in value}}{\text{Original value}} \times 100$
- An increase of 20% means the new value is 120% of the old value
- Percentage decrease = $\frac{\text{decrease in value}}{\text{Original value}} \times 100$
- A decrease of 20% means the new value is 80% of the old value

EXAMPLES

1. Stephen had 60 goats. Now he has 63 goats. What is the percentage increase?

Increase in value = New value - Old value
=
$$63 - 60$$

= 3
Percentage increase = $\frac{\text{increase in value}}{\text{Original value}} \times 100$
= $\frac{3}{60} \times 100$
= 5%

2. The price of bread increased from Shs 3800 to Shs 4000. Find the percentage increase in the price of the item

Increase in value = New value - Old value = 4000 - 3800= 200Percentage increase = $\frac{\text{increase in value}}{\text{Original value}} \times 100$ = $\frac{200}{3800} \times 100$ = 5.263%
3. The price of an item reduced from Shs 8,000 to Shs 6,000. Find the percentage decrease in the price of the item

Decrease in value = Old value - New value = 8000 - 6000= 2000Percentage decrease = $\frac{\text{Decrease in value}}{\text{Original value}} \times 100$ = $\frac{2000}{8000} \times 100$ = 25%

4. An item costing Shs 3,000 is reduced by 20%. Find its new cost price A decrease(reduction) of 20% means the new value is 80% of the old value

New cost price =
$$\frac{80}{100} \times \text{Old value}$$

= $\frac{80}{100} \times 3000$
= Shs2400

EXERCISE

1. The table below shows the marks obtained by some students of s.1 at Taibah international school in two mathematics tests. For each one, calculate the percentage differene(change) and make a conclusion whether it is an increase or a decrease.

Student	Test 1	Test 2	Change in value	Percentage change	Conclusion
Mimmi	92	97	—	—	_
Cooper	100	92	—	—	—
Praise	92	83	—	—	—
Tendo	100	72	—	—	—
Hannah	100	67	—	—	_

- 2. The population of a school increased from 1,200 to 1,500 students. Find the percentage increase in the population of the school
- 3. The price of an item reduced from Shs 4,000 to Shs 3,400. Find the percentage decrease in the price of the item
- 4. In a closing-down sale, a shop offers 50% cut of the original prices. What fraction is taken off the prices?
- 5. In a survey one in five people said they preferred a particular brand of Coca Cola. What is this figure as a percentage?
- 6. Peter pays tax at the rate of 25% of his income. What fraction of Peter's income is this?

- 7. When Carol was buying a house, she had to make a deposit of $\frac{1}{10}$ of the value of the house. What percentage was this?
- 8. I bought a coat in the January sales with $\frac{1}{5}$ price cut of the selling price. What percentage was taken off the price of the coat?
- 9. Akasha bought some fabric that was 1.75 metres long. How could this be written as a fraction?
- 10. A car park contains 20 spaces. There are 17 cars parked in the car park.
 - (a) What Percentage of the car park is full?
 - (b) What Percentage of the car park is empty?
- 11. An item costing Shs 8,000 is increased by 15%. Find its new cost price
- 12. Copy and complete the table. The column headings will help you. You are required to fill in the fraction and percentage columns as done in the first three rows

Tens	Ones	$\operatorname{Tenth}(\frac{1}{10})$	Hundredth $\left(\frac{1}{100}\right)$	Thousandth $\left(\frac{1}{1000}\right)$	Fraction	Percentage
		5			$\frac{1}{2}$	50
1	2	4			$12\frac{2}{5}$	1240
		2	5		$\frac{1}{4}$	25
		1	5	2		
		5				
						80
					$\frac{17}{20}$	
						64
		0	0	4		
					$\frac{3}{10}$	_
4	0	3				
	0	6	4			

ACTIVITY OF INTERGRATION

- Taibah international school has two sections, that is, Lower UNEB (S.1-S.4) and Upper UNEB (S.5-S.6). The Director of studies of the school needs to draw a timetable for the online lessons for both sections. The sections should start and end their morning lessons at the same time before break time, start and end their break time at the same time. The after break lessons should start at the same time. The lunchtime for both sections should start at the same time. The same time and end at the same time. The after Lunch lessons should start at the same time. Math must have 3 hours in a week in each class
- Support: The time to start lessons for the two sections is 8.30am and lessons end at 4:30pm. The duration of the lesson for the Lower UNEB section is 1 hour and that of the Upper UNEB is 2 hours. Assume the following subjects to be offered

Math	English	History	Art	Geography	CRE	Music
Biology	Chemistry	Physics	Entrepreneurship	Home mgt	Psychology	P.E

- Resources: Knowledge of fractions, percentages, natural numbers, factors, multiples, lowest common multiples, and the subjects taught in all classes and of time.
- Tasks:
 - Help the Director of studies by drawing the timetable for the week (monday to friday) for the two sections.
 - How many lessons does each section have up to lunchtime?
 - What is the total number of hours in a week for the lower section

Chapter 4: RECTANGULAR CARTESIAN COORDINATES IN 2 DIMEN-SIONS

Introduction

This topic is key in building the concept of location. The knowledge achieved from this topic can be used in locating places. In order to locate places you need a starting point (reference point).

- A pair of values written in the form (x, y) is called coordinates
- A point with given coordinates can be plotted on the x y plane
- The x y plane is the same as the coordinate plane or the rectangular Cartesian plane
- On the x y plane, the horizontal axis is called the x-axis and the vertical axis is called the y- axis.
- The x-axis meets the x-axis at a point called the origin. The coordinates of the origin are (0, 0)
- On the x-axis, values to the right of the origin are positive and those to the left are negative
- On the y-axis, values above the origin are positive and those below are negative

4.0.1 Identifying the x- axis and y-axis

Activity : Plotting Points **STEPS**:

- Find the value of x on the x- axis.i.e Start from the origin (0,0) and move the required steps along the x- axis
- Locate the value of y on the y- axis.i.e Start from the origin (0,0) and move the required steps along the y- axis
- The intersection of the x and y values is the point

EXAMPLE

Plot the following points on a graph paper A(6,4), B(5,9), C(8,3), D(-4,4), E(-2,-8), F(2,-3), G(3,4), and H(4,-3)

SOLUTION

A(6,5). Start from the origin and first move 6 units to the right (because its positive) ,then 4 units upwards .The intersection is point A



EXERCISE

- 1. (a) Plot the following points on a graph paper A(-4,2), B(-3,5), C(1,5), D(2,2), E(-5,-5), F(-3,-2),G(-1,-5), H(2,-2), I(8,2) , J(8,-4) and K(2,-4)
 - (b) Join points ABCDA, EFGE, HIJKH
 - (c) Name the figures formed in each case



2. (a) Write down the points plotted on the graph paper below



- (b) Join points ABCDA
- (c) Name the figure formed in each case
- 3. Write down the plotted points
 - (a) A_____
 - (b) B_____
 - (c) C_____
 - (d) X_____

- (e) Y _____ (f) T_____
- (g) S_____
- (h) Join points ABC and XYTS ,and name the figures formed in each case



4.0.2 Plotting Polygons (shapes)

A regular polygon is a polygon which is equiangular (all angles are of the same size) and equilateral (all sides have the same length).



EXAMPLE:

1. Join the points A(1,1), B(5,1) and C(3,4) to form a triangle



2. What is the possible coordinate of the corner of the rectangle ABCD?



3. What are the possible coordinates of the corners of the square KLMN?



- 4. In each case the coordinates of 3 corners of a square are given. Find the coordinates of the other corner.
 - (a) (2, -2), (2, 3) and (-3, 3)
 - (b) (2, 3), (3, 4) and (1, 4)
 - (c) (2, 2), (4, 4) and (4, 0)
 - (d) (-6, 2), (-5, -5) and (1, 3)
 - (e) (-5, -2), (-2, -1), and (-1, -4)
- 5. The coordinates of 3 corners of a rectangle are given below. Find the coordinates of the other corner of each rectangle.
 - (a) (-4, 2), (-4, 1) and (6, 1)
 - (b) (0, 2), (-2, 0) and (4, -6)
 - (c) (-4, 5), (-2, -1) and (1, 0)
 - (d) (-5, 1), (-2, 5) and (6, -1)
- 6. (a) The coordinates of 2 corners of a square are (-4, 4) and (1, -1). Explain why it is possible to draw three different squares using these two points.

- (b) Draw the three different squares.
- (c) If the coordinates of the corners had been (-5, 1) and (1, 3) would it still be possible to draw 3 squares? Draw the possible squares.
- 7. Half of a heptagon with one line of symmetry can be drawn by joining the points with coordinates: (2, 4), (-2, 1), (-2, -1), (0, -3) and (2, -3). Join the coordinates. You have drawn one half of the heptagon. Complete the heptagon. Write down the coordinates.
- 8. On the same axes, plot the points P(3, 4), Q(5, 4), R(6, 2) and S(2, 2), Join the points and name the formed figure PQRS.

4.0.3 Use of Appropriate Scale for Given Data

At times we encounter large values for x and y , and for such cases we are required to use a convenient scale such that all our values can be able to fit on the graph paper. EXAMPLE:

Plot the following points on the axes: A(5, 50), B(10,100), C(15,150), D(20,200), E(25,250), F(30, 300), G(35,350).

you realise that on the horizontal axis(x- values) there are 5 units for each space and On the vertical axis (y- values) there are 50 units for each space

Horizontal scale : 1cm:5

Vertical axis :1cm :50



EXERCISE

- 1. For each part, draw a pair of axes with suitable scales and plot the points:
 - (a) A(1, 15), B(4, 35), C(8, 45)
 - (b) M(15, 100), N(35, 500), P(40, 700)

- 2. Plot the points X(2, 60), Y(4, 50), Z(0, 70), T(7, 60)
- 3. On the same axes, plot the following points A(4, 10), B(-2, -40), C(3, 0), D(0, 30), E(-3, 15) and F(0, -20). Use a scale of 1cm to represent 1 unit on the x- axis and 1cm to represent 5 units on the y- axis
- 4. A quadrilateral has vertices A(-10, 0), B(-10, 25), C(15, 25) and D(25, -10). Plot the points of the quadrilateral and identify it. Use a scale of 2cm to represent 10 units on both axes
- 5. A quadrilateral has vertices A(1, 20), B(-3, 30), C(-2, -10) and D(2, -20). Plot the points of the quadrilateral and identify it.

Situation of Integration

A Senior One learner has reported in her class and has settled at her desk.

Support: The classroom is arranged in rows and columns. It is big a big class with each learner having his/ her own desk.

Resources: Knowledge of horizontal and vertical lines i.e. rows and columns, coordinates Knowledge: counting numbers

Task: The mathematics teacher has asked her to explain how she can access her seat, starting from the entrance of the class. Discuss whether there are other ways of reaching her seat.

WISDOM CORNER

- 1. Our greatest weakness lies in giving up. The way to success is trying one more time
- 2. We must accept finite dissappointments but never loose infinite hope
- 3. Procrastination makes easy things hard and hard things harder
- 4. The people who achieve incredible success in the world aren't always the most naturally brilliant but are rather the hardest workers
- 5. The road to success is often a lonely one
- 6. you are braver than you believe, stronger than you seem and smarter than you think
- 7. The more that you read, the more things you will know, the more that you learn, the more places you will go.
- 8. Every successful person has a painful story. Every painful story has a successful ending. Accept the pain and get ready for success
- 9. Successful people have two things on their face silence (to avoid problems) and smile (to solve problems)
- 10. The problem is most people give up during the struggle · Never knowing how close they were to victory!!The only way you'll fail is if you QUIT. NEVER QUIT

REFERENCE: Maths prototype by NCDC

For any inquiries, corrections and additions you can reach me on the following addresses: kazibastephen4@gmail.com 0703822752/0787698238(Whatsapp)

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¹STAY HOME STAY SAFE